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THREE- DIMENSIONAL SIMULATION IN THE SCALAR FUNCTION USING THE SIMPLEX ELEMENT

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Abstract. The finite element method is a numerical method for solving differential equations encountered in physics and technology. The emergence of this method is associated with the solution of space research problems. It was first published in the work of Turner, Cluj, Martin and Topp. This work contributed to the emergence of other works; a number of articles were published with examples of the finite element method to the problems of structural mechanics of continuous media.

The main idea of the finite element method is that any continuous quantity, such as temperature, pressure and displacement, can be approximated by a discrete model, which is built on a set of piece-continuous functions defined on a finite number of subdomains.

The finite element method has evolved from a numerical procedure for solving problems in structural mechanics into a general method for numerically solving a differential equation or a system of differential equations. This progress has been made over a fifteen-year period through the development of high-speed, digital computing machines needed for more accurate calculations of aircraft structures, as well as through "the assistance of the National Committee for Space Research. The computing machine has accelerated many complex numerical calculations. space required the allocation of funds for fundamental research and stimulated the improvement of universal computing programs.The finite element method is used in the design of aircraft, rockets, various spatial shells.

Key words: Finite element, pipe, motion, compression, lubrication, system, vibration, equilibrium, continuous value, discrete model, cross section, node.

Introduction

In the general case, the continuous quantity is unknown in advance, and it is necessary to determine the value of this quantity at some interior points of the region. A discrete model, however, is very easy to construct if we first assume that the numerical values of this quantity at each interior point of the region are known. After that, you can move on to the general case. For each element, its own polynomial is determined, but the polynomials are selected in such a way that the continuity of the value along the boundaries of the element is preserved.

The finite element method is based on the idea of approximating a continuous function by a discrete model, which is built on the set of piecewise continuous functions defined on a finite number of subdomains, called elements. A polynomial is most often used as an element function. The order of the polynomial depends on the number of continuous function data items used at each node.

Main part

A one-dimensional simplex element is a straight line segment of length L with two nodes, one at each end of the segment.

$$T = \varphi_1 T_1 + \varphi_2 T_2$$

$$\varphi_1 = \frac{x_2 - x}{L} \qquad \varphi_2 = \frac{x - x_1}{L}$$

A two-dimensional simplex element is a triangle with rectilinear sides and three nodes, one for each vertex requires logical numbering of element nodes.

$$T = \varphi_1 T_1 + \varphi_2 T_2 + \varphi_3 T_3$$

$$\varphi_1 = \frac{1}{2A}(a_1 + b_1 x + c_1 y)$$

$$\begin{cases} a_1 = x_2 y_3 - x_3 y_2 \\ b_1 = y_2 - y_3 \\ c_1 = x_3 - x_2 \end{cases}$$

$$\varphi_2 = \frac{1}{2A}(a_2 + b_2 x + c_2 y)$$

$$\begin{cases} a_2 = x_3 y_1 - y_3 y_1 \\ b_2 = y_3 - y_1 \\ c_2 = x_1 - x_3 \end{cases}$$

$$\varphi_3 = \frac{1}{2A}(a_3 + b_3 x + c_3 y)$$

$$\begin{cases} a_3 = x_1 y_2 - x_2 y_1 \\ b_3 = y_1 - y_2 \\ c_3 = x_2 - x_1 \end{cases}$$

$$2A = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

We will consider a three-dimensional simplex element.

$$T = \varphi_1 T_1 + \varphi_2 T_2 + \varphi_3 T_3 + \varphi_4 T_4$$

Consider a three-dimensional function T (x, y, z), the value of which is given at the corner points of the parallelepiped. (1,2, ... 8) (1-T, 2-T, ... 8-T) (Fig. 1)

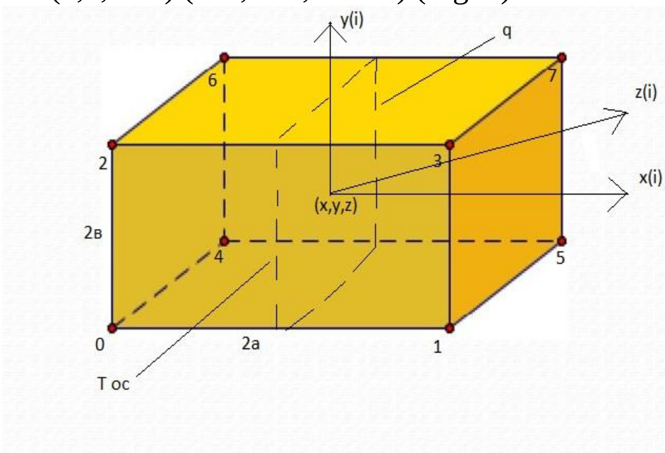

Figure 1- parallelepiped

If the length of the parallelepiped along the x→ 2a, along the y→ 2b, and along the z→ 2c, then the coordinates of the corner points relative to the center of the parallelepiped will be.

1- (-a; в; -c)= (x_1, y_1, z_1)
2- (a; в; -c)= (x_2, y_2, z_2)
3- (a; -в; -c)= (x_3, y_3, z_3)
4- (-a; -в; -c)= (x_4, y_4, z_4)
5- (-a; в; c)= (x_5, y_5, z_5)
6- (a; в; c)= (x_6, y_6, z_6)
7- (a; -в; c)= (x_7, y_7, z_7)
8-(-a; -в; c) = (x_8, y_8, z_8)

The value of the function T (x, y, z) to an arbitrary point (x, y, z) inside the parallelepiped is approximated as follows.

$$T(x,y,z) = \lambda_1 + \lambda_2 x_1 + \lambda_3 y_1 + \lambda_4 z_1 + \lambda_5 x_1 y_1 + \lambda_6 x_1 z_1 + \lambda_7 y_1 z_1 + \lambda_8 x_1 y_1 z_1 \qquad (1)$$

To determine the constants Λ_i (i = 1; 8), we compose the following control system:

$$T_1 = T(x_1,y_1,z_1);\ T_2 = (x_2,y_2,z_2);\ T_3 = (x_3,y_3,z_3);$$
$$T_4 = T(x_4,y_4,z_4);\ T_5 = T(x_5,y_5,z_5);\ T_6 = T(x_6,y_6,z_6); \qquad (2)$$
$$T_7 = T(x_7,y_7,z_7);\ T_8 = T(x_8,y_8,z_8);$$

Substituting the values of the arguments, we get the system of equations:

$$\begin{cases} \lambda_1 + \lambda_2 x_1 + \lambda_3 y_1 + \lambda_4 z_1 + \lambda_5 x_1 y_1 + \lambda_6 x_1 z_1 + \lambda_7 y_1 z_1 + \lambda_8 x_1 y_1 z_1 = T_1 \\ \lambda_1 + \lambda_2 x_2 + \lambda_3 y_2 + \lambda_4 z_2 + \lambda_5 x_2 y_2 + \lambda_6 x_2 z_2 + \lambda_7 y_2 z_2 + \lambda_8 x_2 y_2 z_2 = T_2 \\ \lambda_1 + \lambda_2 x_3 + \lambda_3 y_3 + \lambda_4 z_3 + \lambda_5 x_3 y_3 + \lambda_6 x_3 z_3 + \lambda_7 y_3 z_3 + \lambda_8 x_3 y_3 z_3 = T_3 \\ \lambda_1 + \lambda_2 x_4 + \lambda_3 y_4 + \lambda_4 z_4 + \lambda_5 x_4 y_4 + \lambda_6 x_4 z_4 + \lambda_7 y_4 z_4 + \lambda_8 x_4 y_4 z_4 = T_4 \\ \lambda_1 + \lambda_2 x_5 + \lambda_3 y_5 + \lambda_4 z_5 + \lambda_5 x_5 y_5 + \lambda_6 x_5 z_5 + \lambda_7 y_5 z_5 + \lambda_8 x_5 y_5 z_5 = T_5 \\ \lambda_1 + \lambda_2 x_6 + \lambda_3 y_6 + \lambda_4 z_6 + \lambda_5 x_6 y_6 + \lambda_6 x_6 z_6 + \lambda_7 y_6 z_6 + \lambda_8 x_6 y_6 z_6 = T_6 \\ \lambda_1 + \lambda_2 x_7 + \lambda_3 y_7 + \lambda_4 z_7 + \lambda_5 x_7 y_7 + \lambda_6 x_7 z_7 + \lambda_7 y_7 z_7 + \lambda_8 x_7 y_7 z_7 = T_7 \\ \lambda_1 + \lambda_2 x_8 + \lambda_3 y_8 + \lambda_4 z_8 + \lambda_5 x_8 y_8 + \lambda_6 x_8 z_8 + \lambda_7 y_8 z_8 + \lambda_8 x_8 y_8 z_8 = T_8 \end{cases} \qquad (3)$$

Solving this system of linear equations, we obtain the values of the coefficients $\lambda_1, \lambda_2 \ldots \lambda_8$

$$\begin{cases} \Lambda_1 = \frac{T_8+T_7+T_6+T_5+T_4+T_3+T_2+T_1}{8} \\ \Lambda_2 = \frac{-T_8+T_7+T_6-T_5-T_4+T_3+T_2-T_1}{8a} \\ \Lambda_3 = \frac{-T_8-T_7+T_6+T_5-T_4-T_3+T_2+T_1}{8b} \\ \Lambda_4 = \frac{T_8+T_7+T_6+T_5-T_4-T_3-T_2-T_1}{8c} \\ \Lambda_5 = \frac{T_8+T_7+T_6+T_5+T_4+T_3+T_2+T_1}{8ab} \\ \Lambda_6 = \frac{T_8-T_7+T_6-T_5+T_4-T_3+T_2-T_1}{8ac} \\ \Lambda_7 = \frac{-T_8-T_7+T_6+T_5+T_4+T_3-T_2-T_1}{8bc} \\ \Lambda_8 = \frac{T_8-T_7+T_6-T_5-T_4+T_3-T_2+T_1}{8abc} \end{cases}$$

(7) substituting these values into equation (2) we obtain:

$$T(x,y,z) = \varphi_1(x,y,z) * T_1 + \varphi_2(x,y,z) * T_2 + \varphi_3(x,y,z) * T_3 + \varphi_4 T_4 + \varphi_5(x,y,z) * T_5 + \varphi_6(x,y,z) * T_6 + \varphi_7(x,y,z) * T_7 + \varphi_8(x,y,z) * T_8; \qquad (4)$$

-a ≤ x ≤a; -b ≤x ≤b; -c ≤ x≤ c;

Here φ_i(i= 1, 8) are defined as follows:

$$\begin{aligned} \varphi_1(x,y,z) &= \left(\frac{1}{8} - \frac{x}{8a} + \frac{y}{8b} - \frac{z}{8c} - \frac{xy}{8ab} + \frac{xz}{8ac} - \frac{yz}{8bc} + \frac{xyz}{8abc}\right); \\ \varphi_2(x,y,z) &= \left(\frac{1}{8} + \frac{x}{8a} + \frac{y}{8b} - \frac{z}{8c} + \frac{xy}{8ab} - \frac{xz}{8ac} - \frac{yz}{8bc} - \frac{xyz}{8abc}\right); \\ \varphi_3(x,y,z) &= \left(\frac{1}{8} + \frac{x}{8a} - \frac{y}{8b} - \frac{z}{8c} - \frac{xy}{8ab} - \frac{xz}{8ac} + \frac{yz}{8bc} + \frac{xyz}{8abc}\right); \\ \varphi_4(x,y,z) &= \left(\frac{1}{8} - \frac{x}{8a} - \frac{y}{8b} - \frac{z}{8c} + \frac{xy}{8ab} + \frac{xz}{8ac} + \frac{yz}{8bc} - \frac{xyz}{8abc}\right); \\ \varphi_5(x,y,z) &= \left(\frac{1}{8} - \frac{x}{8a} + \frac{y}{8b} + \frac{z}{8c} - \frac{xy}{8ab} - \frac{xz}{8ac} + \frac{yz}{8bc} - \frac{xyz}{8abc}\right); \\ \varphi_6(x,y,z) &= \left(\frac{1}{8} + \frac{x}{8a} + \frac{y}{8b} + \frac{z}{8c} + \frac{xy}{8ab} + \frac{xz}{8ac} + \frac{yz}{8bc} + \frac{xyz}{8abc}\right); \\ \varphi_7(x,y,z) &= \left(\frac{1}{8} + \frac{x}{8a} - \frac{y}{8b} + \frac{z}{8c} - \frac{xy}{8ab} + \frac{xz}{8ac} - \frac{yz}{8bc} - \frac{xyz}{8abc}\right); \\ \varphi_8(x,y,z) &= \left(\frac{1}{8} - \frac{x}{8a} - \frac{y}{8b} + \frac{z}{8c} + \frac{xy}{8ab} - \frac{xz}{8ac} - \frac{yz}{8bc} + \frac{xyz}{8abc}\right); \end{aligned} \qquad (5)$$

why -a ≤ x ≤a; -b ≤y ≤b; -c ≤ z≤ c

The value of the functions φ_i(x, y, z) (i=1,8) at the corner points of the parallelepiped is determined as follows:

$$\varphi_1(x_1,y_1,z_1)=1;\ \varphi_1(x_1,y_1,z_1)=\varphi_1(x_2,y_2,z_2)=\varphi_1(x_3,y_3,z_3)=\varphi_1(x_4,y_4,z_4)=\varphi_1(x_5,y_5,z_5)=\varphi_1(x_6,y_6,z_6)=\varphi_1(x_7,y_7,z_7)=\varphi_1(x_8,y_8,z_8)=0$$
$$\varphi_2(x_2,y_2,z_2)=1;\ \varphi_2(x_1,y_1,z_1)=\varphi_2(x_2,y_2,z_2)=\varphi_2(x_3,y_3,z_3)=\varphi_2(x_4,y_4,z_4)=\varphi_2(x_5,y_5,z_5)=\varphi_2(x_6,y_6,z_6)=\varphi_2(x_7,y_7,z_7)=\varphi_2(x_8,y_8,z_8)=0$$
$$\varphi_3(x_3,y_3,z_3)=1;\ \varphi_3(x_1,y_1,z_1)=\varphi_3(x_2,y_2,z_2)=\varphi_3(x_3,y_3,z_3)=\varphi_3(x_4,y_4,z_4)=\varphi_3(x_5,y_5,z_5)=\varphi_3(x_6,y_6,z_6)=\varphi_3(x_7,y_7,z_7)=\varphi_3(x_8,y_8,z_8)=0$$
$$\varphi_4(x_4,y_4,z_4)=1;\ \varphi_4(x_1,y_1,z_1)=\varphi_4(x_2,y_2,z_2)=\varphi_4(x_3,y_3,z_3)=\varphi_4(x_4,y_4,z_4)=\varphi_4(x_5,y_5,z_5)=\varphi_4(x_6,y_6,z_6)=\varphi_4(x_7,y_7,z_7)=\varphi_4(x_8,y_8,z_8)=0 \quad (6)$$
$$\varphi_5(x_5,y_5,z_5)=1;\ \varphi_5(x_1,y_1,z_1)=\varphi_5(x_2,y_2,z_2)=\varphi_5(x_3,y_3,z_3)=\varphi_5(x_4,y_4,z_4)=\varphi_5(x_5,y_5,z_5)=\varphi_5(x_6,y_6,z_6)=\varphi_5(x_7,y_7,z_7)=\varphi_5(x_8,y_8,z_8)=0$$
$$\varphi_6(x_6,y_6,z_6)=1;\ \varphi_6(x_1,y_1,z_1)=\varphi_6(x_2,y_2,z_2)=\varphi_6(x_3,y_3,z_3)=\varphi_6(x_4,y_4,z_4)=\varphi_6(x_5,y_5,z_5)=\varphi_6(x_6,y_6,z_6)=\varphi_6(x_7,y_7,z_7)=\varphi_6(x_8,y_8,z_8)=0$$
$$\varphi_7(x_7,y_7,z_7)=1;\ \varphi_7(x_1,y_1,z_1)=\varphi_7(x_2,y_2,z_2)=\varphi_7(x_3,y_3,z_3)=\varphi_7(x_4,y_4,z_4)=\varphi_7(x_5,y_5,z_5)=\varphi_7(x_6,y_6,z_6)=\varphi_7(x_7,y_7,z_7)=\varphi_7(x_8,y_8,z_8)=0$$
$$\varphi_8(x_8,y_8,z_8)=1;\ \varphi_8(x_1,y_1,z_1)=\varphi_8(x_2,y_2,z_2)=\varphi_8(x_3,y_3,z_3)=\varphi_8(x_4,y_4,z_4)=\varphi_8(x_5,y_5,z_5)=\varphi_8(x_6,y_6,z_6)=\varphi_8(x_7,y_7,z_7)=\varphi_8(x_8,y_8,z_8)=0$$

Now let's calculate the temperature gradient within the volume of one parallelepiped:

$$\frac{\partial T}{\partial x}=\sum_{i=1}^{8}\frac{\partial\varphi_i}{\partial x}T_i,\ \mathrm{i}=1,8;\ \frac{\partial T}{\partial y}=\sum_{i=1}^{8}\frac{\partial\varphi_i}{\partial y}T_i;\ \frac{\partial T}{\partial z}=\sum_{i=1}^{8}\frac{\partial\varphi_i}{\partial z}T_i \quad (7)$$

We calculate separately $\frac{\partial\varphi_i}{\partial x}$:

$$\frac{\partial\varphi_1}{\partial x}=(-\frac{1}{8a}-\frac{y}{8ab}+\frac{z}{8ac}+\frac{yz}{8abc});$$
$$\frac{\partial\varphi_1}{\partial y}=(\frac{1}{8b}-\frac{x}{8ab}-\frac{z}{8bc}+\frac{xz}{8abc});$$
$$\frac{\partial\varphi_1}{\partial z}=(-\frac{1}{8c}+\frac{x}{8ac}-\frac{y}{8bc}+\frac{xy}{8abc});$$
$$\frac{\partial\varphi_2}{\partial x}=(\frac{1}{8a}+\frac{y}{8ab}-\frac{z}{8ac}-\frac{yz}{8abc});$$
$$\frac{\partial\varphi_2}{\partial y}=(\frac{1}{8b}+\frac{x}{8ab}-\frac{z}{8bc}-\frac{xz}{8abc});$$
$$\frac{\partial\varphi_2}{\partial z}=(-\frac{1}{8c}-\frac{x}{8ac}-\frac{y}{8bc}-\frac{xy}{8abc});$$
$$\frac{\partial\varphi_3}{\partial x}=(\frac{1}{8a}-\frac{y}{8ab}-\frac{z}{8ac}+\frac{yz}{8abc});$$
$$\frac{\partial\varphi_3}{\partial y}=(-\frac{1}{8b}-\frac{x}{8ab}+\frac{z}{8bc}+\frac{xz}{8abc});$$
$$\frac{\partial\varphi_3}{\partial z}=(-\frac{1}{8c}-\frac{x}{8ac}+\frac{y}{8bc}+\frac{xy}{8abc});$$
$$\frac{\partial\varphi_4}{\partial x}=(-\frac{1}{8a}+\frac{y}{8ab}+\frac{z}{8ac}-\frac{yz}{8abc});$$
$$\frac{\partial\varphi_4}{\partial y}=(-\frac{1}{8b}+\frac{x}{8ab}+\frac{z}{8bc}-\frac{xz}{8abc}); \quad (8)$$
$$\frac{\partial\varphi_4}{\partial z}=(-\frac{1}{8c}+\frac{x}{8ac}+\frac{y}{8bc}-\frac{xy}{8abc});$$
$$\frac{\partial\varphi_5}{\partial x}=(-\frac{1}{8a}-\frac{y}{8ab}-\frac{z}{8ac}-\frac{yz}{8abc});$$
$$\frac{\partial\varphi_5}{\partial y}=(\frac{1}{8b}-\frac{x}{8ab}+\frac{z}{8bc}-\frac{xz}{8abc});$$
$$\frac{\partial\varphi_5}{\partial z}=(\frac{1}{8c}-\frac{x}{8ac}+\frac{y}{8bc}-\frac{xy}{8abc});$$
$$\frac{\partial\varphi_6}{\partial x}=(\frac{1}{8a}+\frac{y}{8ab}+\frac{z}{8ac}+\frac{yz}{8abc});$$
$$\frac{\partial\varphi_6}{\partial y}=(\frac{1}{8b}+\frac{x}{8ab}+\frac{z}{8bc}+\frac{xz}{8abc});$$
$$\frac{\partial\varphi_6}{\partial z}=(\frac{1}{8c}+\frac{x}{8ac}+\frac{y}{8bc}+\frac{xy}{8abc});$$
$$\frac{\partial\varphi_7}{\partial x}=(\frac{1}{8a}-\frac{y}{8ab}+\frac{z}{8ac}-\frac{yz}{8abc});$$

$\frac{\partial\varphi_7}{\partial y}= (-\frac{1}{8b}-\frac{x}{8ab}-\frac{z}{8bc}-\frac{xz}{8abc});$

$\frac{\partial\varphi_7}{\partial z}= (\frac{1}{8c}+\frac{x}{8ac}-\frac{y}{8bc}-\frac{xy}{8abc});$

$\frac{\partial\varphi_8}{\partial x}= (-\frac{1}{8a}+\frac{y}{8ab}-\frac{z}{8ac}+\frac{yz}{8abc});$

$\frac{\partial\varphi_8}{\partial y}= (-\frac{1}{8b}+\frac{x}{8ab}-\frac{z}{8bc}+\frac{xz}{8abc});$

$\frac{\partial\varphi_8}{\partial z}= (\frac{1}{8c}-\frac{x}{8ac}-\frac{y}{8bc}+\frac{xy}{8abc});$

$$T(x,y,z) = \varphi_1 T_1 + \varphi_2 T_2 + \varphi_3 T_3 + \varphi_4 T_4 + \varphi_5 T_5 + \varphi_6 T_6 + \varphi_7 T_7 \varphi_8 T_8; \quad (9)$$

$$\frac{\partial T}{\partial x} = \frac{\partial\varphi_1}{\partial x}T_1 + \frac{\partial\varphi_2}{\partial x}T_2 + \frac{\partial\varphi_3}{\partial x}T_3 + \frac{\partial\varphi_4}{\partial x}T_4 + \frac{\partial\varphi_5}{\partial x}T_5 \frac{\partial\varphi_6}{\partial x}T_6 \frac{\partial\varphi_7}{\partial x}T_7) + \frac{\partial\varphi_8}{\partial x}T_8;$$
$$\frac{\partial T}{\partial y} = \frac{\partial\varphi_1}{\partial y}T_1 + \frac{\partial\varphi_2}{\partial y}T_2 + \frac{\partial\varphi_3}{\partial y}T_3 + \frac{\partial\varphi_4}{\partial y}T_4 + \frac{\partial\varphi_5}{\partial y}T_5 \frac{\partial\varphi_6}{\partial y}T_6 \frac{\partial\varphi_7}{\partial y}T_7) + \frac{\partial\varphi_8}{\partial y}T_8; \quad (10)$$
$$\frac{\partial T}{\partial z} = \frac{\partial\varphi_1}{\partial z}T_1 + \frac{\partial\varphi_2}{\partial z}T_2 + \frac{\partial\varphi_3}{\partial z}T_3 + \frac{\partial\varphi_4}{\partial z}T_4 + \frac{\partial\varphi_5}{\partial z}T_5 \frac{\partial\varphi_6}{\partial z}T_6 \frac{\partial\varphi_7}{\partial z}T_7) + \frac{\partial\varphi_8}{\partial z}T_8;$$

$$F(0) = \left(\frac{1}{8}-\frac{x}{8a}-\frac{y}{8b}--\frac{z}{8c}+\frac{xy}{8ab}+\frac{xz}{8ac}+\frac{yz}{8bc}-\frac{xyz}{8abc}\right);$$
$$F(1) = \left(\frac{1}{8}+\frac{x}{8a}-\frac{y}{8b}-\frac{z}{8c}-\frac{xy}{8ab}-\frac{xz}{8ac}+\frac{yz}{8bc}+\frac{xyz}{8abc}\right);$$
$$F(2) = \left(\frac{1}{8}-\frac{x}{8a}-\frac{y}{8b}-\frac{z}{8c}-\frac{xy}{8ab}-\frac{xz}{8ac}+\frac{yz}{8bc}+\frac{xyz}{8abc}\right);$$
$$F(3) = \left(\frac{1}{8}+\frac{x}{8a}-\frac{y}{8b}-\frac{z}{8c}+\frac{xy}{8ab}+\frac{xz}{8ac}+\frac{yz}{8bc}-\frac{xyz}{8abc}\right);$$
$$F(4) = \left(\frac{1}{8}+\frac{x}{8a}+\frac{y}{8b}+\frac{z}{8c}-\frac{xy}{8ab}-\frac{xz}{8ac}+\frac{yz}{8bc}-\frac{xyz}{8abc}\right); \quad (11)$$
$$F(5) = \left(\frac{1}{8}-\frac{x}{8a}+\frac{y}{8b}+\frac{z}{8c}+\frac{xy}{8ab}+\frac{xz}{8ac}+\frac{yz}{8bc}+\frac{xyz}{8abc}\right);$$
$$F(6) = \left(\frac{1}{8}+\frac{x}{8a}-\frac{y}{8b}+\frac{z}{8c}-\frac{xy}{8ab}+\frac{xz}{8ac}-\frac{yz}{8bc}-\frac{xyz}{8abc}\right);$$
$$F(7) = \left(\frac{1}{8}-\frac{x}{8a}-\frac{y}{8b}+\frac{z}{8c}+\frac{xy}{8ab}-\frac{xz}{8ac}-\frac{yz}{8bc}+\frac{xyz}{8abc}\right);$$

$$F(x,y,z) = F(0)T_0 + F(1)T_1 + F(2)T_2 + F(3)T_3 + F(4)T_4 + F(5)T_5 + F(6)T_6 + F(7)T_7;$$

The general functional is

$$J = J_1 + J_2 + J_3;$$
$$J_1 = \int_V \frac{1}{2}[k_{xx}(\frac{\partial T}{\partial x})^2 + k_{yy}(\frac{\partial T}{\partial y})^2 + k_{zz}(\frac{\partial T}{\partial z})^2]dv;$$
$$J_2 = \int_{S_n} qT^{'}ds;$$
$$J_3 = \int_{S_n} \frac{h}{2}(T^2 - T_{oc})^2 ds;$$
$$T^1 = F(4)T_4 + F(5)T_5 + F(6)T_6 + F(7)T_7;$$
$$T^2 = F(0)T_0 + F(1)T_1 + F(2)T_2 + F(3)T_3;$$
$$J_1 = \int_{-a}^{a}\int_{-b}^{b}\int_{-c}^{c} k_{xx}[(\frac{\partial T(x,y,z)}{\partial x})^2 + (\frac{\partial T(x,y,z)}{\partial y})^2 + (\frac{\partial T(x,y,z)}{\partial z})^2]dxdydz;$$
$$J_2 = \int_{-a}^{a}\int_{-b}^{b} qT^{'}(x,y)dxdy;$$
$$J_3 = \int_{-a}^{a}\int_{-b}^{b} \frac{h}{2}(T^2 - T_{oc})^2 dxdy;$$
$$J = J_1 + J_2 + J_3 = J(T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7);$$

$$\begin{cases} \frac{\partial J()}{\partial T_0} = 0, \\ \frac{\partial J()}{\partial T_1} = 0, \\ \ldots \\ \frac{\partial J()}{\partial T_7} = 0, \end{cases} \quad \begin{cases} a_{00}T_0 + a_{01}T_1 + a_{02}T_2 + a_{03}T_3 + a_{04}T_4 + a_{05}T_5 + a_{06}T_6 + a_{07}T_7 = b_0 \\ a_{10}T_0 + a_{11}T_1 + a_{12}T_2 + a_{13}T_3 + a_{14}T_4 + a_{15}T_5 + a_{16}T_6 + a_{17}T_7 = b_1 \\ \ldots \\ a_{70}T_0 + a_{71}T_1 + a_{72}T_2 + a_{73}T_3 + a_{74}T_4 + a_{75}T_5 + a_{76}T_6 + a_{77}T_7 = b_7 \end{cases};$$

Result: $T^{'}_0, T^{'}_1, T^{'}_2, T^{'}_3, T^{'}_4, T^{'}_5, T^{'}_6, T^{'}_7;$

$$T(x,y,z) = F(0)T^{'}_0 + F(1)T^{'}_1 + F(2)T^{'}_2 + F(3)T^{'}_3 + F(4)T^{'}_4 + F(5)T^{'}_5 + F(6)T^{'}_6 + F(7)T^{'}_7;$$

$x = 0;$

$T(y,z) = F(0)T'_0 + F(1)T'_1 + F(2)T'_2 + F(3)T'_3 + F(4)T'_4 + F(5)T'_5 + F(6)T'_6 + F(7)T'_7;$

$y = 0$'

Conclusion

The results of this work can be used to determine the temperature distribution law in three-dimensional rods in the form of a parallelepiped. When solving problems by the finite element method, a variety of elements are used. Some of the more important ones were introduced in this chapter in connection with the consideration of solid body discretization. These elements are emphasized for several reasons. They are simple in theory, which makes it easy to illustrate their application. Triangular and tetrahedral elements can be used to approximate complex boundaries because they can be oriented as desired. Another important reason is that many of the available computing programs use these elements.

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