UDC 681.511.46
IRSTI 28.15.00

STABILIZATION AND OPTIMUM CONTROL OF TWO-MACHINE SYSTEM

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Abstract. The article deals with the problem of optimal motion control of a two-machine system. The problems of stabilization and control of a two-machine system are described by nonlinear differential equations. These mathematical models describe the processes in complex systems consisting of many turbines and generators, and are used to analyze them. The relevance of these models lies in the fact that they allow you to model various pre-emergency, emergency and post-emergency situations. The stability of the synthesized system is checked by the Lyapunov function method. The correctness of the solutions found is verified by the numerical solution of the considered and given example.

The controllability of the model under consideration is determined by the study of the global asymptotic stability of dynamical systems in cylindrical phase systems. The results obtained are demonstrated by a numerical example.

Keywords: phase system, synchronous generator, steam turbine, Euler's method, optimal control, stability, method of Lyapunov function, numerical method.

Introduction

Consider a simplified model of the "synchronous generator – steam turbine" system described by differential equations of the form for the two-machine case:

$$\frac{d\delta_i}{dt} = S_i$$

$$T_{ji}\frac{dS_i}{dt} = P_{Ti} - K_i S_i - \left[\frac{E^2}{z_{11}}\sin\alpha_{11i} + \frac{EU_i}{z_{12}}\sin(\delta_i - \alpha_{12i})\right]$$

$$T_{Pi}\frac{dP_{Ti}}{dt} = -P_{Ti} + \rho_{0i}P_{0i} - \frac{P_{0i}}{\sigma_{0i}}S_i + u.$$

P_{Ti} – steam turbine power; T_{Pi} – time constant of the control cycle of the steam turbine; ρ_{0i}, P_{0i} – given constant values; σ_{0i} – statism of ASC (automatic speed controller); δ_i – EMF angle of the generator; S_i – generator slip; T_i – constant inertia of moving masses; $K_i > 0$ – damping coefficient; E_i – calculated EMF of the generator; U_i – voltage on tires of infinite power; z_{11i} – intrinsic resistance of the generator; z_{12i} – mutual resistance between the generator and tires; α_{11i} – additional angle of intrinsic resistance; α_{12i} – additional angle of mutual resistance.

Let the following parameters of the steam turbine control system be given:

$T_P = 251.2$, $\rho_0 = 0.994$, $P_0 = 10420$, $\sigma_0 = 0.06$

Carrying out the transfer of the origin of coordinates to the equilibrium position $(\sigma, S, P_T) = (0.686, 0, 10357.48)$, we pass to the system of equations of the perturbed motion:

$$\frac{d\delta_1}{dt} = S_1, \quad \frac{d\delta_2}{dt} = S_2$$
$$\frac{dS_1}{dt} = C_1 P_1 - K_1 S_1 - \bar{f}_1(\delta_1)$$
$$\frac{dS_2}{dt} = C_2 P_2 - K_2 S_2 - \bar{f}_2(\delta_2) \tag{1}$$
$$\frac{dP_1}{dt} = -\bar{A}_1 P_1 + u_1, \quad \frac{dP_2}{dt} = -\bar{A}_2 P_2 + u_2.$$

where $C_1 = 4.167*10^{-8}$, $C_2 = 4.167*10^{-8}$, $K_1 = 66.7*10^{-4}$, $K_2 = 66.7*10^{-4}$, $f_0 = 1.513*10^{-4}$, $\theta_0 = 0.3562$, $A_1 = 39.81*10^{-4}$, $A_2 = 39.81*10^{-4}$, $f_i(\delta_i) = f_0\left[\sin(\delta_i + \theta_0) - \sin\theta_0\right]$.

System (1) can be rewritten as:

$$\frac{dx}{dt} = A(t)x + B(t)u + f(t,u,x), \quad t \in [t_0, t_1], \quad x(t_0) = x_0.$$

where

$$A(t) = A = \begin{vmatrix} 0 & 1 & 0 \\ 0 & -K & C \\ 0 & 0 & \bar{A} \end{vmatrix}, \quad B(t) = B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$x(t) = \begin{bmatrix} \delta(t) \\ S(t) \\ P_T(t) \end{bmatrix}, \quad f(x,u,t) = f(x) = \begin{bmatrix} 0 \\ -\bar{f}(\delta) \\ 0 \end{bmatrix}.$$

as

$$\left|\bar{f}(\delta') - \bar{f}(\delta'')\right| - f_0\left|\sin(\delta' + \theta_0) - \sin(\delta'' + \theta_0)\right| \le f_0\left|\delta' - \delta''\right|,$$

then the function f(x) satisfies the Lipschitz condition:

$$\left|f(t,u^1,x^1) - f(t,u^2,x^2)\right| \le L_1\left|u^1 - u^2\right| + L_2\left|x^1 - x^2\right|,$$
$$\left(\forall u^1, u^2 \in \Omega \subseteq R^r, \forall x^1, x^2 \in G \subseteq R^n\right)$$

where $L_1 = 0, L_2 = 2f_0$. Linear stationary system:

$$\frac{dy}{dt} = Ay + Bv, \quad t \in [t_0, t_1], \quad y(t_0) = x_0, \quad \left(y \in R^n, v \in R^r\right) \tag{2}$$

completely controllable as the rank of the Kalman matrix

$$U=\left(B,AB,A^2B\right)=\begin{pmatrix}0 & 0 & C\\ 0 & C & -\bar{A}C\\ 1 & -\bar{A} & \bar{A}^2\end{pmatrix}$$ is equal to 3.

We calculate the fundamental matrix (2). The characteristic matrix $\lambda E_3 - A$ will be equal to:

$$\lambda E_3 - A=\begin{pmatrix}\lambda & -1 & 0\\ 0 & \lambda+K & -C\\ 0 & 0 & \lambda+\bar{A}\end{pmatrix}$$ and characteristic determinant: $\Delta(\lambda)=\det(\lambda E_3 - A)=\lambda(\lambda+K)(\lambda+\bar{A})$.

The matrix attached to the matrix $\lambda E_3 - A$ has the form:

$$ad_j\left(\lambda E_3 - A\right)=\begin{pmatrix}(\lambda+K)(\lambda+\bar{A}) & \lambda+\bar{A} & C\\ 0 & \lambda(\lambda+\bar{A}) & C\lambda\\ 0 & 0 & \lambda(\lambda+K)\end{pmatrix}$$

Common greatest divisor of the elements of the adjoint matrix: $D_2(\lambda)=1$. The minimum polynomial of matrix A will be as follows:

$$\psi(\lambda)=\frac{\Delta(\lambda)}{D_2(\lambda)}=\lambda(\lambda+K)(\lambda+\bar{A}).$$

The values of the function $e^{\lambda t}$ on the spectrum of the matrix A will be

$$\left(e^{\lambda t}\right)_{\lambda=0}=1,\ \left(e^{\lambda t}\right)_{\lambda=-K}=e^{-Kt},\ \left(e^{\lambda t}\right)_{\lambda=-\bar{A}}=e^{-\bar{A}t}.$$

The interpolation conditions are as follows: $r(\lambda_1)=1$, $r(\lambda_2)=e^{-Kt}$, $r(\lambda_3)=e^{-\bar{A}t}$. Where $\lambda_1=0$, $\lambda_2=-K$, $\lambda_3=-\bar{A}$ – the roots of the characteristic equation $\det(\lambda E_3 - A)=0$.

Since for the considered matrix A:

$$\psi_1(\lambda)=\frac{\psi(\lambda)}{\lambda-\lambda_1}=(\lambda+K)(\lambda+\bar{A}),$$

$$\psi_2(\lambda)=\frac{\psi(\lambda)}{\lambda-\lambda_2}=\lambda(\lambda+\bar{A}),$$

$$\psi_3(\lambda)=\frac{\psi(\lambda)}{\lambda-\lambda_3}=\lambda(\lambda+K)$$

Then the Lagrange-Sylvester interpolation polynomial [4] has the form

$$r(\lambda)=\left(\frac{e^{\lambda t}}{(\lambda+K)(\lambda+\bar{A})}\right)_{\lambda=0}(\lambda+K)(\lambda+\bar{A})+\left(\frac{e^{\lambda t}}{\lambda(\lambda+\bar{A})}\right)_{\lambda=-K}\lambda(\lambda+\bar{A})+$$
$$+\left(\frac{e^{\lambda t}}{\lambda(\lambda+K)}\right)_{\lambda=-\bar{A}}\lambda(\lambda+K)=1+\beta_1(t)\lambda+\beta_2(t)\lambda^2$$

where $\beta_1(t)=\frac{K+\bar{A}}{K\bar{A}}-\frac{\bar{A}e^{-Kt}}{K(\bar{A}-K)}-\frac{Ke^{-\bar{A}t}}{\bar{A}(K-\bar{A})}$, $\beta_2(t)=\frac{1}{K\bar{A}}-\frac{e^{-Kt}}{K(\bar{A}-K)}-\frac{e^{-\bar{A}t}}{\bar{A}(K-\bar{A})}$.

Therefore,

$$e^{At}=E_3+\beta_1(t)A+\beta_2(t)A^2=\begin{pmatrix}1 & \beta_1-K\beta_2 & \beta_2 C\\ 0 & 1-K\beta_1+K^2\beta_2 & \beta_2 C-\bar{A}C\beta_2\\ 0 & 0 & 1-\bar{A}\beta_1+\bar{A}^2\beta_2\end{pmatrix}.$$

why,

$$\beta_1-K\beta_2=\frac{\bar{A}}{K\bar{A}}-\frac{\bar{A}e^{-Kt}}{K(\bar{A}-K)}+\frac{e^{-Kt}}{\bar{A}-K}=\frac{1}{K}\left(1-e^{-Kt}\right)$$

$(\beta_1-\bar{A}\beta_2)C=\frac{C}{\bar{A}}\left(1-e^{-\bar{A}t}\right)$, $1-K\beta_1+K^2\beta_2=e^{-Kt}$, $1-\bar{A}\beta_1+\bar{A}^2\beta_2=e^{-\bar{A}t}$.

Then we finally have:

$$\theta(t)=e^{At}\begin{pmatrix}1 & \frac{1}{K}\left(1-e^{-Kt}\right) & r_0-r_1e^{-Kt}+r_2e^{-\bar{A}t}\\ 0 & e^{-Kt} & \frac{C}{\bar{A}}\left(1-e^{-\bar{A}t}\right)\\ 0 & 0 & e^{-\bar{A}t}\end{pmatrix}.$$

where, $r_0=\frac{C}{K\bar{A}}$, $r_1=\frac{C}{K(\bar{A}-K)}$, $r_2=\frac{C}{\bar{A}(\bar{A}-K)}$. If $t_0=0$, then:

$\Phi(t,t_0)=\theta(t)$, $\Phi(t_0,t)=\theta^{-1}(t)$, $\theta(t_0)=E_3$, $\theta^{-1}(t_0)=E_3$ and $W(0,t)=\int_0^t e^{A(t-r)}BB^*e^{A^*(t-r)}dr$,

$W(0,t_1)=\int_0^{t_1} e^{A(t_1-r)}BB^*e^{A^*(t_1-r)}dr$.

In our case, the equations are written as follows:

$$\tilde{\delta}_i=\delta_{i-1}+hf(\delta_{i-1}),$$
$$\delta_i=\delta_{i-1}+h\frac{f(\delta_{i-1})+f(\tilde{\delta}_i)}{2},$$
$$\tilde{S}_i=S_{i-1}+hf(\delta_{i-1},S_{i-1},P_{i-1}),$$

$$S_i = S_{i-1} + h\frac{f\left(\delta_{i-1}, S_{i-1}, P_{i-1}\right) + f\left(\tilde{\delta}_i, \tilde{S}_i, \tilde{P}_i\right)}{2},$$

$$\tilde{P}_i = P_{i-1} + hf\left(P_{i-1}\right),$$

$$P_i = P_{i-1} + h\frac{f\left(P_{i-1}\right) + f\left(\tilde{P}_{i-1}\right)}{2}.$$

It is also easy to calculate the inverse matrix $\theta^{-1}(t)$:

$$\theta^{-1}(t) = e^{At} = \begin{pmatrix} 1 & \frac{1}{K}\left(1 - e^{Kt}\right) & l_1\left(e^{\bar{\lambda}_1 t} - e^{Kt}\right) - l_2 e^{\bar{A}t} + r_1 e^{\lambda_2 t} + l_3 \\ 0 & e^{Kt} & l_4\left(e^{Kt} - e^{\bar{\lambda}_1 t}\right) \\ 0 & 0 & e^{-\bar{A}t} \end{pmatrix}$$

Numerical calculation.

Predictor: $\tilde{y}_i = y_{i-1} + hf\left(x_{i-1}, y_{i-1}\right)$. Corrector: $y_i = y_{i-1} + h\frac{f\left(x_{i-1}, y_{i-1}\right) + f\left(x_i, \tilde{y}_i\right)}{2}$.
The results are shown below:

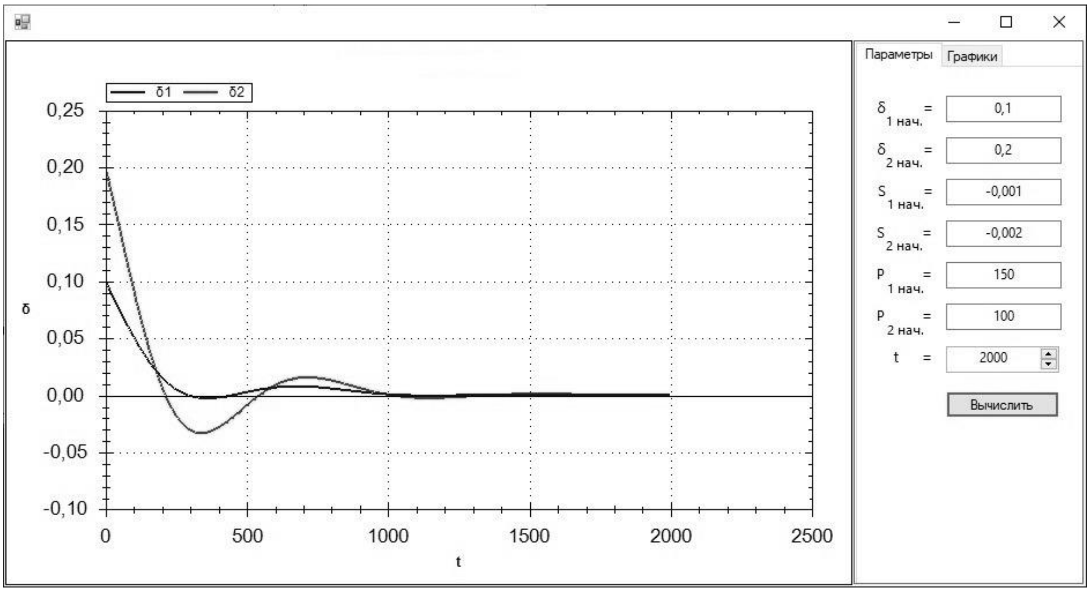

Figure 1 - δ1, δ2

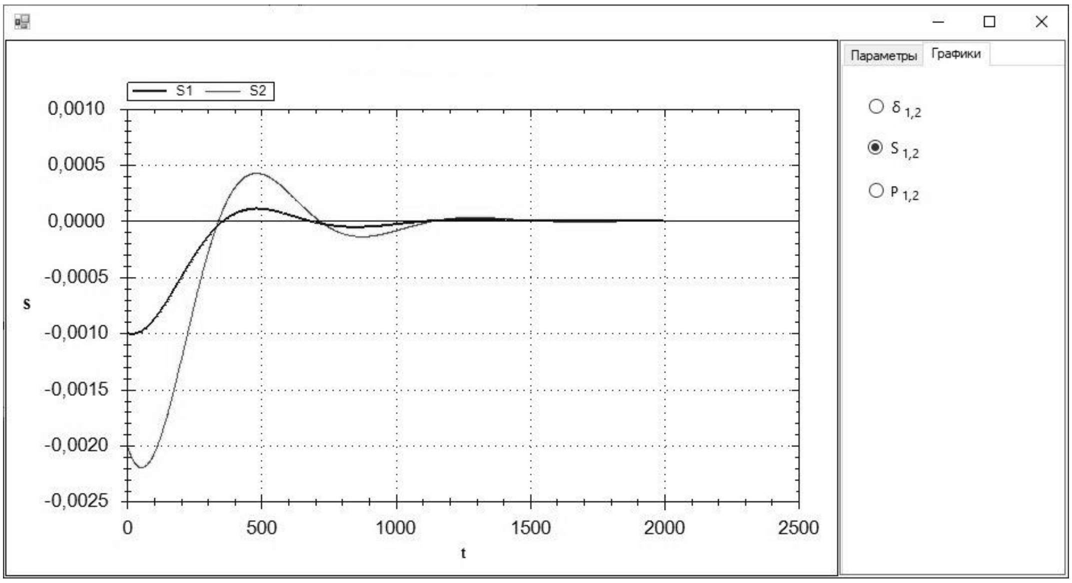

Figure 2 - S1, S2

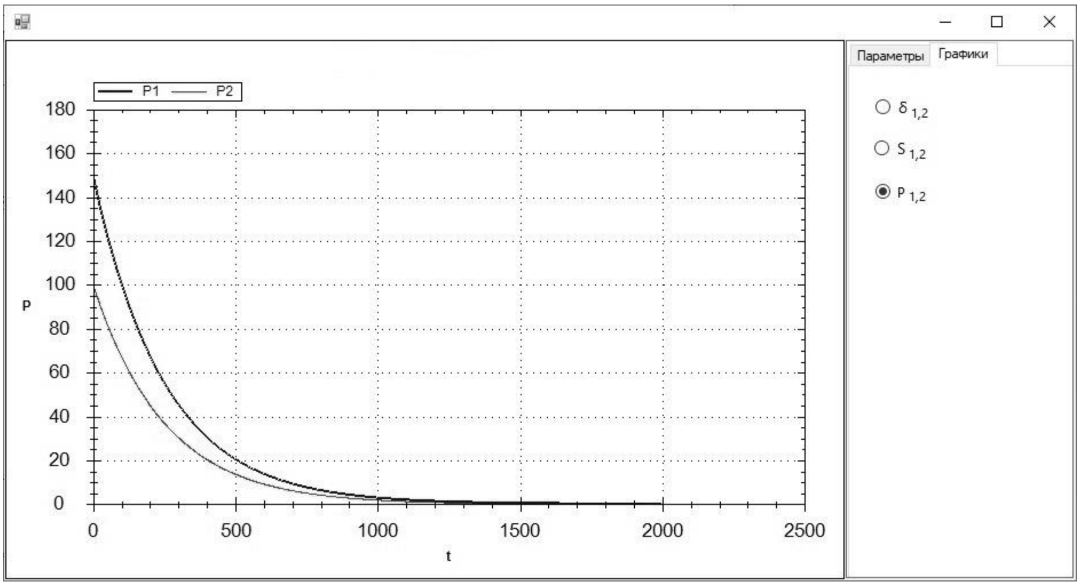

Figure 3 - P1, P2

Consider the problem of optimal motion control of two-machine system. The stability of the synthesized system is tested by the Lyapunov function method. The correctness of the solutions found is verified by the numerical solution of the considered and the example.

One of the mathematical models that describes transient processes in a two-machine electrical system is the following system of differential equations:

$$\frac{d\delta_1}{dt} = S_1$$

$$\frac{d\delta_2}{dt} = S_2 \tag{3}$$

$$H_1 \frac{dS_1}{dt} = -E_1^2 Y_{11} \sin\alpha_{11} - P_1 \sin\left(\delta_1 - \alpha_1\right) - P_{12} \sin\left(\delta_{12} - \alpha_{12}\right) + u_1$$

$$H_2 \frac{dS_2}{dt} = -E_2^2 Y_{22} \sin\alpha_{22} - P_2 \sin\left(\delta_2 - \alpha_2\right) - P_{21} \sin\left(\delta_{21} - \alpha_{21}\right) + u_2$$

$\delta_{12} = \delta_1 - \delta_2$, $\delta_{21} = \delta_2 - \delta_1$, $P_1 = E_1 U Y_{1,n+1}$, $P_{12} = E_1 E_2 Y_{12}$

where δ_i – the angle of rotation of the rotor of the i-th generator relative to some synchronous axis of rotation, S_i – slip of the i-th generator, H_i – constant of inertia of the i-th machine; $u_i = P_{Ti}$ – mechanical power supplied to the generator; E_i – EMF of the i-th synchronous machine, $U = const$ – DC bus voltage; $Y_{1,n+1}$ – characterizes the connection of the i-th generator with constant voltage buses; Y_{ij} – mutual conductivity of the i-th and j-th branches of the system; $\alpha_{ii}, \alpha_i, \alpha_{ij}$ – constant values that take into account the effect of active resistances in the stator circuits of generators; $\alpha_{ij} = \alpha_{ji}$.

Let the state variables and control in the steady-state post-emergency mode have the following meanings:

Si = 0, $\delta_i = \delta_i^F$, $u_i = u_i^F$, i = 1, 2.

Perturbed motion equations:

$$\begin{aligned} &\frac{d\delta_1}{dt} = S_1 \\ &\frac{dS_1}{dt} = \frac{1}{H_1}\left[-f_1(\delta_1) - N_1(\delta) - M_1(\delta) + u_1\right] \\ &\frac{d\delta_2}{dt} = S_2 \\ &\frac{dS_2}{dt} = \frac{1}{H_2}\left[-f_2(\delta_2) - N_1(\delta) - M_1(\delta) + u_2\right] \end{aligned} \quad (4)$$

where

$f_1(\delta_1) = P_1\left[\sin\left(\delta_1 + \delta_1^F - \alpha_1\right) - \sin\left(\delta_1^F - \alpha_1\right)\right]$,

$f_2(\delta_2) = P_2\left[\sin\left(\delta_2 + \delta_2^F - \alpha_2\right) - \sin\left(\delta_2^F - \alpha_2\right)\right]$,

$N_1(\delta) = \Gamma_1\left[\sin\left(\delta_{12} + \delta_{12}^F\right)\right]$

$M_1(\delta) = \Gamma_2\left[\sin\left(\delta_{12} + \delta_{12}^F\right)\right]$

where $\delta_{12}^F = \delta_1^F - \delta_2^F$, $\Gamma_1 = P_{12}\cos\alpha_{12}$, $\Gamma_2 = P_{12}\sin\alpha_{12}$.

Numerical data of the system:

$\delta_1 = -0.052$, $\delta_2 = -0.104$, H_1=2135, H_2=1256, P_1=0.85, P_2=0.69, δ_1^F =0.827, δ_2^F =0.828, $\alpha_{12} = -0.078$ and initial conditions:

$\delta_1(0) = 0.18$, $\delta_2(0) = 0.1$, $S_1(0) = 0.001$, $S_2(0) = 0.002$.

Consider the following optimal control problem: minimize functionality

$$J(v) = J(v_1, ..., v_l) = \frac{1}{2}\int_0^T \sum_{i=1}^{l}\left(w_{si}S_i^2 + w_{vi}v_i^2\right)dt + \Lambda\left(\delta(T)S(T)\right) \quad (5)$$

under conditions:

$$\frac{d\delta_i}{dt} = S_i$$
$$\frac{dS_i}{dt} = \frac{1}{H_i}\left[-f_i(\delta_i) - N_i(\delta) - M_i(\delta) + \nu_i\right], \quad (6)$$
$i = \overline{1,l}$, $t \in [0,T]$, $\delta = (\delta_1,...,\delta_l)$, $S = (S_1,...,S_l)$

where $w_{si}, w_{\nu i}$ – positive constant weight coefficients; $f_i(\delta_i)$ – 2π – periodic continuously differentiable function; $N_i(\delta)$ – 2π – periodic continuously differentiable function with respect to δ_{ij}; with respect to the term $N_i(\delta)$ the integrability condition is satisfied

$$\frac{\partial N_i(\delta)}{\partial \delta_k} = \frac{\partial N_k(\delta)}{\partial \delta_i} \qquad (\forall i \neq k) \quad (7)$$

T – the duration of the transient is considered unknown.

Given initial conditions are:

$$\delta_i(0) = \delta_{i0},\ S_i(0) = S_{i0},\ \ i = \overline{1,l} \quad (8)$$

and $\delta(T),\ S(T)$ unknown in advance.

Theorem 1. For the control $\nu_i^0(S_i) = -\frac{1}{w_{\nu i}} S_i$, $i = \overline{1,l}$ and the corresponding solution of system (6) - (8) to be optimal, it is necessary and sufficient that $\Lambda(\delta(T),S(T)) = K(\delta(T),S(T))$ and $w_{Si} = 2D_i + \frac{1}{w_{\nu i}} > 0,\ \ i = \overline{1,l}$ where

$$K(\delta,S) = \frac{1}{2}\sum_{i=1}^{l} H_i S_i^2 + \sum_{i=1}^{l}\int_0^{\delta_i} f_i(\delta_i)d\delta_i + \sum_{i=1,\delta_j=0,\ j>i}^{l}\int_0^{\delta_i} N_i(\delta_1,...,\delta_{i-1},\xi_i,\delta_{i+1},...,\delta_l)\,d\xi_i$$

– Bellman function, at that

$$J(\nu^0) = \min_{\nu} J(\nu) = K(\delta_0, S_0). \quad (9)$$

Evidence

For a continuous function $K(\delta(t),S(t))$ of the variable t-functional (5) can be represented as:

$$J(\nu) = J(\delta(t),S(t),\nu(t)) = \int_0^T R(\delta(t),S(t),\nu(t))dt + m_0(\delta(0),S(0)) + m_i(\delta(T)S(T))$$

where

$$R(\delta,S,\nu)=\sum_{i=1}^{l}\left[K_{\delta i}S_i+\frac{1}{H_i}K_{Si}\left(D_iS_i-f_i(\delta_i)-N_i(\delta)+f_i(\delta_i)+\nu_i\right)+\frac{1}{2}\left(w_{Si}S_i^2+w_{\nu i}\nu_i^2\right)\right] \quad (10)$$

where

$$K_{\delta i}=-\frac{\partial K}{\partial \delta_i},\ K_{Si}=-\frac{\partial K}{\partial S_i}. \quad (11)$$

To determine the Bellman function $K(\delta,S)$, consider the following Cauchy-Bellman problem:

$$\inf_{\nu} R(\delta,s,\nu)=0,\quad 0\le t\le T,\ K(\delta(T),S(T))=\Lambda(\delta(T),S(T)). \quad (12)$$

From the necessary condition for the extremum of the function $R(\delta,S,\nu)$ with respect to $\nu_i\in R_i^1$ we obtain

$$R_{\nu i}\equiv\frac{1}{H_i}K_{Si}+w_{\nu i}\nu_i=0,\quad i=\overline{1,l},$$

Therefore the optimal controls

$$\nu_i^0\equiv-\frac{1}{H_iw_{\nu i}}K_{Si},\ i=\overline{1,l}. \quad (13)$$

The function $K(\delta,S)$ and the weight coefficients $w_{si},w_{\nu i}$ are found from condition (12) i.e:

$$\bar{R}=\min_{\nu}R(\delta,S,\nu)=\sum_{i=1}^{l}\left[K_{\delta i}S_i-\frac{1}{H_i}K_{Si}\left(D_iS_i+f_i(\delta_i)+N_i(\delta)\right)-\frac{1}{2H_i^2}K_{Si}^2+\frac{1}{2}w_{Si}S_i^2\right]=0 \quad (14)$$

For this we put $K_{\delta i}S_i=\frac{K_{Si}}{H_i}\left(f_i(\delta_i)+N_i(\delta)\right),\ i=\overline{1,l}$ i.e.

$$K_{Si}=H_iS_i,\ K_{\delta i}=f_i(\delta_i)+N_i(\delta),\ i=\overline{1,l}.$$

Then, taking into account (15), from relation (14) we obtain that

$$\sum_{i=1}^{l}\left[-D_iS_i^2-\frac{1}{w_{\nu i}}S_i^2+\frac{1}{2}w_{Si}S_i^2\right]=0$$

or

$$w_{Si}=2D_i+\frac{1}{2w_{\nu i}}>0,\quad w_{\nu i}>0,\ i=\overline{1,l}. \quad (15)$$

Moreover, from (13) we obtain that the optimal controls $\nu_i^0,\ i=\overline{1,l}$ have the form:

$$v_i^0\left(S_i\right)=-\frac{1}{w_{vi}}S_i,\quad i=\overline{1,l}. \qquad (16)$$

Let us now consider the question of how the Bellman function – $K_{Si},\ K_{\delta i}$ can be determined, knowing the quotients $K\left(\delta,S\right)$. The integrability conditions(15) for the function $K\left(\delta,S\right)$ are equivalent to the condition (7). Really

$$\frac{\partial N_i\left(\delta\right)}{\partial\delta_k}=-\Gamma_{ik}^1\cos\left(\delta_{ik}+\delta_{ik}^F\right),$$
$$\frac{\partial N_k\left(\delta\right)}{\partial\delta_i}=-\Gamma_{ki}^1\cos\left(\delta_{ki}+\delta_{ki}^F\right)=-\Gamma_{ik}^1\cos\left(\delta_{ik}+\delta_{ik}^F\right).$$

Consequently, the function $K\left(\delta,S\right)$ can be represented in the form:

$$K\left(\delta,S\right)=\frac{1}{2}\sum_{i=1}^{l}H_iS_i^2+\sum_{i=1}^{l}\int_0^{\delta_i}f_i\left(\delta_i\right)d\delta_i+\sum_{\substack{i=1,\\ \delta_j=0,\,j>i}}^{l}\int_0^{\delta_i}N_i\left(\delta_1,...,\delta_{i-1},\xi_i,\delta_{i+1},...,\delta_i\right)d\xi_i=$$
$$=\frac{1}{2}\sum_{i=1}^{l}H_iS_i^2+\sum_{i=1}^{l}\int_0^{\delta_i}f_i\left(\delta_i\right)d\delta_i+\int_0^{\delta_1}\sum_{j=1,\,j\neq i}^{l}\bar{N}_{1j}\left(\xi_1,0,...0\right)d\xi_1+$$
$$=\int_0^{\delta_2}\sum_{j=1,\,j\neq i}^{l}\bar{N}_{2j}\left(\delta_1,\xi_2,0,...0\right)d\xi_2+...+\int_0^{\delta_l}\sum_{j=1,\,j\neq i}^{l}\bar{N}_{lj}\left(\delta_1,...,\delta_{l-1},\xi_l\right)d\xi_2. \qquad (17)$$

Note that for the case l=2:

$$\int_0^{\delta_1}N_{12}\left(\xi_1,0\right)d\xi_1+\int_0^{\delta_2}\bar{N}_{21}\left(\delta_1,\xi_2\right)d\xi_2=\Gamma_{12}^1\left[-\cos\left(\delta_1+\delta_{12}^F\right)-\delta_1\sin\delta_{12}^F+\cos\delta_{12}^F\right]+$$

$$+\Gamma_{21}^1\left[-\cos\left(\delta_{12}+\delta_{12}^F\right)+\delta_2\sin\delta_{12}^F+\cos\left(\delta_1+\delta_{12}^F\right)\right]=\Gamma_{12}^1\left[-\cos\left(\delta_{12}+\delta_{12}^F\right)-\delta_{12}\sin\delta_{12}^F+\cos\delta_{12}^F\right].$$

On the other hand

$$\int_0^{\delta_{12}}N_{12}\left(\delta\right)d\delta=\int_0^{\delta_{12}}\Gamma_{12}^1\left[\sin\left(x+\delta_{12}^F\right)-\sin\delta_{12}^F\right]dx=\Gamma_{21}^1\left[-\cos\left(\delta_{12}+\delta_{12}^F\right)+\delta_{12}\sin\delta_{12}^F+\cos\delta_{12}^F\right].$$

therefore, for l = 2, as well as for any l> 2, the function $K\left(\delta,S\right)$ from (17) can be represented in the form:

$$K\left(\delta,S\right)=\frac{1}{2}\sum_{i=1}^{l}H_iS_i^2+\sum_{i=1}^{l}\int_0^{\delta_i}f_i\left(\delta_i\right)d\delta_i+\sum_{j=2}^{l}\sum_{i=1}^{j-1}\int_0^{\delta_{ij}}N_{ij}\left(x\right)dx.$$

Observe, that in the region $\{\delta, S\}$, where $K(\delta, S)$ – is a definitely positive function, the Bellman function $K(\delta, S)$ becomes the Lyapunov function for the synthesized system (6), (16) i.e. the synthesized system is Lyapunov stable. According to the boundary condition, for the Bellman equation (12), we can take the function $\Lambda(\delta, S)$ in the form

$$\Lambda(\delta(T), S(T)) = K(\delta(T), S(T))$$

and the value of the functional $J(v)$, will be equal to the value (9). The theorem is proved. According to the considered theorem, the optimal control has the form:

$$u_1 = -\frac{1}{w_1} S_1 - M_1(\delta),$$

$$u_2 = -\frac{1}{w_2} S_2 - M_2(\delta),$$

where $w_1 = 0.1$, $w_2 = 0.1$.

The results of the numerical solution of the equation are given below:

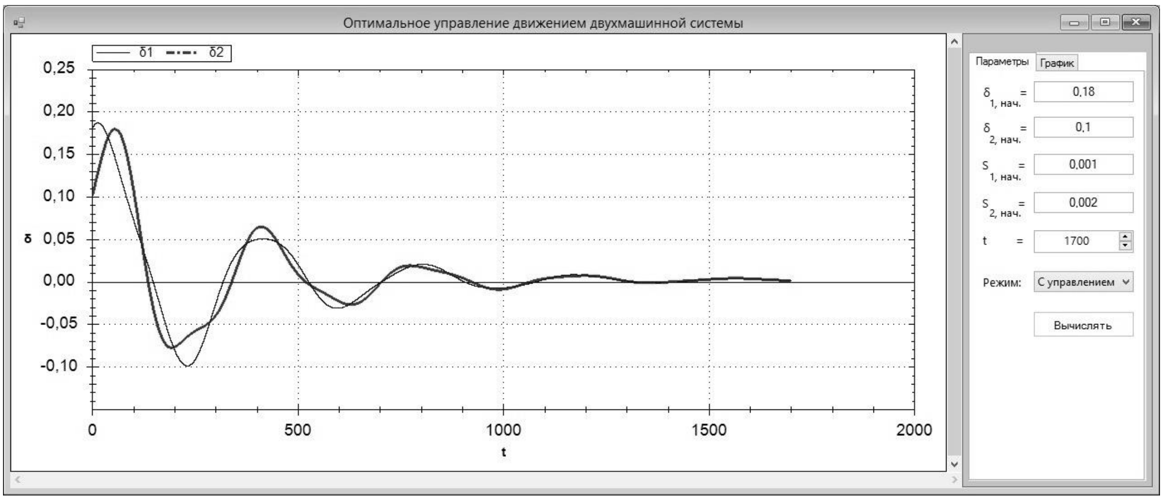

Figure 4 - δ1, δ2

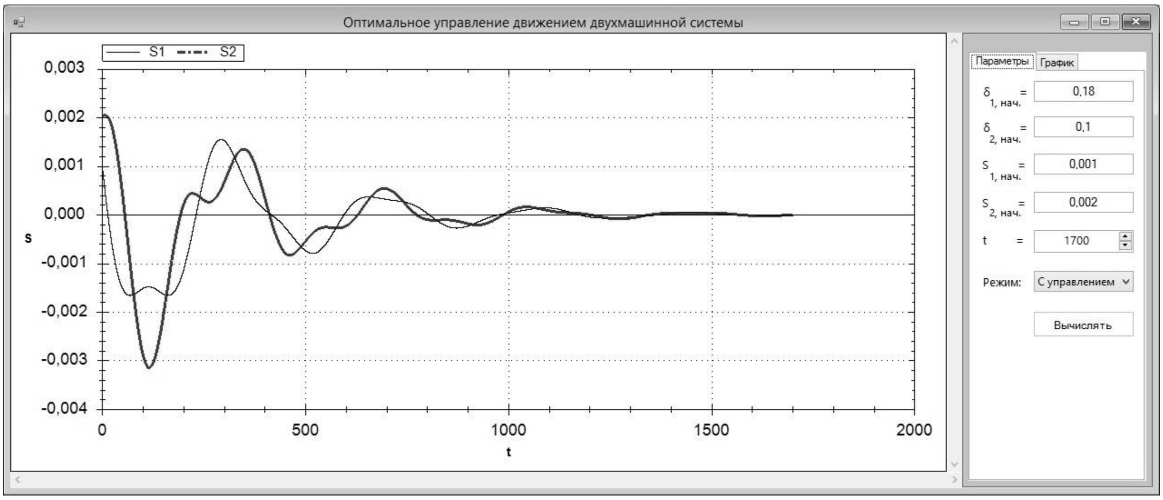

Figure 5 - S1, S2

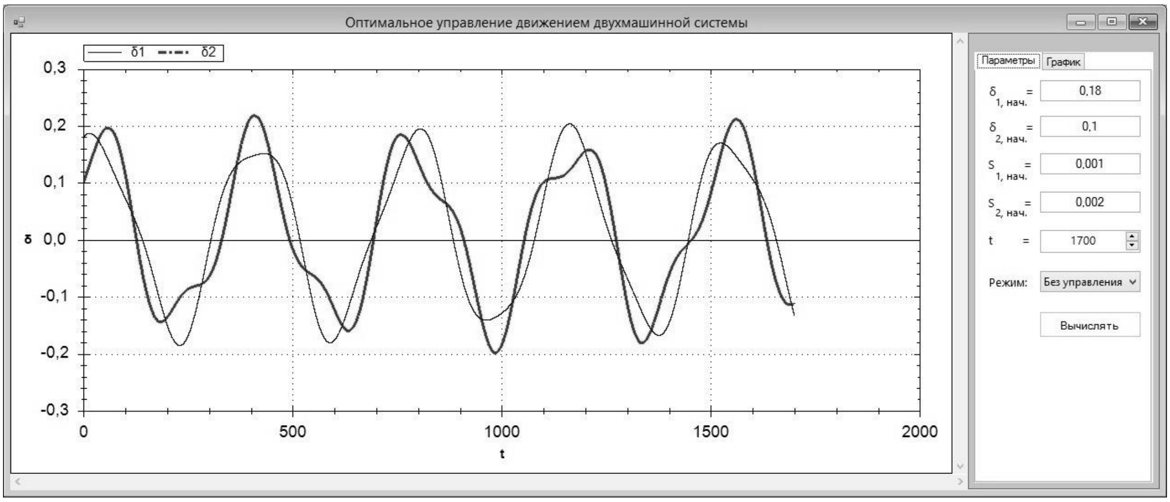

Figure 6 - δ1, δ2 – without control.

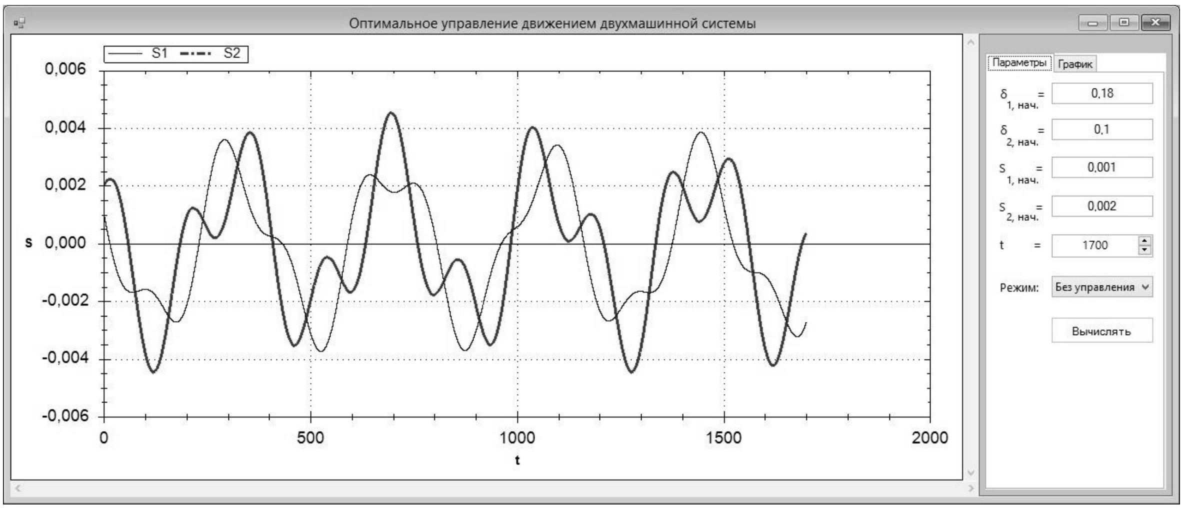

Figure 7 - S1, S2 – without control.

Conclusion

To check the accuracy of the obtained results created programs. Programs written in the C# programming language. For the numerical solution of the task is used modified Euler method. As can be seen from the received graphs, the received controls ensure the stable operation of the system by minimizing the deviation of the system parameters from the primary values. And in case of lack of control the parameters of the system are deviated from the initial value and the system does not work stably.

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