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INTRODUCING PQI-OPERATOR IN THEORY OF COMPUTATIONAL COMPLEXITY

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Abstract. In this article we describe the notion of PQI-operator used before in application role for implementing extended finite state automatons within the subset construction; we give precise definition of this operator and prove important complexity theorems according to the model provided and prior work; the presented operation method can be used for the estimation of the overridden complexity as per automatons constructed through modified subset construction which was well studied before. The complexity, thus, can be of different type according to the classification which was presented earlier in this research of the decidability of complex algorithms which are impractical due to the state explosion, for example, in Rabin-Scott subset construction. These algorithms' complexity can be the type of factorial also. We will show that there exists a PQI-operator followed from the approach presented in the cycle of these works, thus, this work can be seen as the continuation of the study of the theory of automatons and state complexity.

Keywords: computational complexity, operator, theoretical computer science, subset construction, finite state automatons.

Introduction

To the present time the operators defined in theory of complexity weren't studied from the point of view when this complexity is lower than the power-set and, at the same time, continues to expand to the proper linear function. We introduce the new operator named as PQI for the estimation of algorithms with artificial noise which lowers the upper bound of the given consumptions like data and source function – this function is of combinatorial type and expands very fast as exponential.

We give the notion to the algorithms of combinatorial complexity in big O-notation [1] – this is a common and simple way of describing the number of operations for the given data size.

The power-set in this work is present due to the work of finite state automatons [2]. The complexity itself can be computable P or not computable NP [3]. Within the subset construction explosion property was observed before [4].

To our observations we note that the power-set explosion in subset construction follows from the trivial cases which can be as rare and frequent at the same time.

We will further also show by the plotting the convergional difference of PQI-operator set of functions to the set of combinatorial functions which are NP-complete and are, thus, impractical as is, however, the different models and solutions to these problems were provided before in recent research papers and the machine learning trend where neural network can result in variable expectation with the stated problem of huge amount of input data to be trained – of course, we know that this is a disputable question, at least, author of this work finds it as impractical as the collected or obtained data are of practical point of view to get the qualified set of data within the necessary level, so that the proposed neural network gives the expected and adequate result.

As this is a conceptually latest author's work for the question of equality, inequality or decidability of P and NP classes, we won't give the broad description of the theory of automata from which the sought operator was devised – it can be well studied in [5].

In this section we give the plan of our seminal work: first we describe the PQI-operator itself, then we give the notion for the model of operational calculus. The practical results are also presented.

PQI-operator. We define the PQI-operator from the observations presented in the prior work [5], where the subset construction was overridden by the extension states in finite state automatons.

This function according to the estimation of the subset construction for explosion of number of states can be described as follows:

$$2^N - 2^{\frac{N}{2}} , \tag{1}$$

as it naturally follows from (1) that the estimated complexity of subset construction [5] is less than the common case, it's necessary to introduce the arbitrary function p(x) and reduce the complexity class to the normal when it's growing faster than the linear function f(x) = x:

$$\frac{2^N}{p(x)}, p(x) - linear \tag{2}$$

The derivative of the expression (1) is defined in (3):

$$[\ln(2) \cdot 2^x] - p'(x) \tag{3}$$

whereas the derivative of the expression (2) is as follows:

$$\frac{(\ln(2) \cdot 2^N \cdot p(x) - p'(x) \cdot 2^N)}{[p^2(x)]} \tag{4}$$

We define the limit for the expression (4) to be converged to infinity as the complexity is measured from this range, thus we get the following with respect to the L'Hôpital's rule:

$$\lim_{x\to\infty}\left(\frac{p'(x) \cdot 2^x}{p^2(x)}\right) = \lim_{x\to\infty}\left(\frac{p''(x) \cdot 2^x + \ln(2) \cdot 2^x \cdot p'(x)}{2 \cdot p(x) \cdot p'(x)}\right). \tag{5}$$

The equation (5) is measured according to the O(n) complexity notation and equals to the following result:

$$\lim_{x\to\infty}\frac{O(p''(x)) \cdot 2^x + 2^x \cdot p'(x)}{O(2 \cdot p(x) \cdot p'(x))} : p(x) \leqslant p'(x) = \frac{O(2^x)}{P(2 \cdot p(x))} = \frac{O(2^x)}{O(p(x))}. \tag{6}$$

Consequently, it follows that the relation (6) complies to the fact that the function p(x) exists in the power-set space from complexity measure:

$$\lim_{x\to\infty} O(2^x) < \lim_{x\to\infty} O(p(x)) : \exists\, p(x). \tag{7}$$

We give the notion to the comparable complex functions like power-set of the variable power value which is greater than one, thus giving us the increasing function:

$$2^N - 1.8^N. \tag{8}$$

For the set of combinatorial functions of NP nature we state that the derivative of these functions is equal among complexity classes like P and NP:

$$f(x) = f'(x), f(x) = \{2^x, x!\}. \tag{9}$$

The derivative of the discrete combinatorial function like factorial is defined as follows:

$$(x!)' = (x-1)\cdot(x-1)!. \tag{10}$$

From the facts (9) and (10) it follows that the derivatives of the combinatorial functions like power-set and factorial are equal along the complexity and, thus, converge to the NP-class:

$$O(f(x)) = O(f'(x)) : f(x) = \{2^x, x!\}. \tag{11}$$

For the PQI-operator we define the class of functions when the complexity is less when converging to the operational point like infinity – from this side it's possible to estimate the exponential growth:

$$\left(\frac{2}{t}\right)^x : t \leq 2 \rightarrow O(f(x)) \neq O(p(x)), x \rightarrow infinity. \tag{12}$$

Finally for the big O-notation we define the PQI operator as follows:

$$PQI(x) = \frac{2^x}{p(x)} = \frac{2^x}{2^{\log(x)}} = 2^{\frac{x}{\log(x)}} = 2^{\log(x)} : O(p(x)) < O(2^x). \tag{13}$$

The differential equations (10) and (11) prove the assumption (13).

Application of PQI-model. In [5] we have developed the overridden model for the modified subset construction which is relevant to the reduced cost of computation, thus, giving better results.

We have also developed the Java application package “Regex+” for the extended regular expressions.

The PQI-model in this package is successfully implemented via overridden PQI-operator and tagging rules giving the correct results for all the test cases provided within.

The implemented solution isn't, thus, recursive and combinatorially measured resulting to the class of effective algorithms where NP-completeness is avoided – this fact conventionally holds true for better evaluation, as we can represent the typical finite state machine as a neural network for the type of graph structures.

However, this graph structure is specific and, as described before, has the different and complex notation rather than typical finite state automatons – we will call these state of automatons as PQI, or PQI-automata along the provided and evaluated empirical results.

The proof was given also in prior works for the extended operators in regular expression and corresponding finite state automaton, as this proof is essential in giving the definition of PQI-operator and, similarly, PQI-model, which are different from the point of view of estimation and measurement and vice-versa of the practical experience.

In turn, the product construction machines, which where studied in modern works, give not quadratic, but exponential growth for the number of states in the resulting deterministic automaton – however, with our introduction and concept of PQI-operator we obtain results which are minimalistic which can be proven from the facts provided in the equlibria (1)–(14).

More facts about the evaluation of this empirical and proved model show our interest in developing even mobile-aware applications which can be obtained from author of this work by request.

For better discussion of the presented PQI-model we can state that P versus NP problem, which is still an open question, can be reduced to the empirical PQI-model and give the linear growth reduction of the exponentially growing complexity.

In-depth study. In this section the graphical plot of PQI-operator for the variety of combinatorial functions which are not limited to factorial and power-set.

These graphical plots from the initial values show that, the PQI-operator still is less relevant to the combinatorial function derivatives – thus, to be more proper, it's non-convergent and is equally convergent along the size of input data when they are limited to infinity.

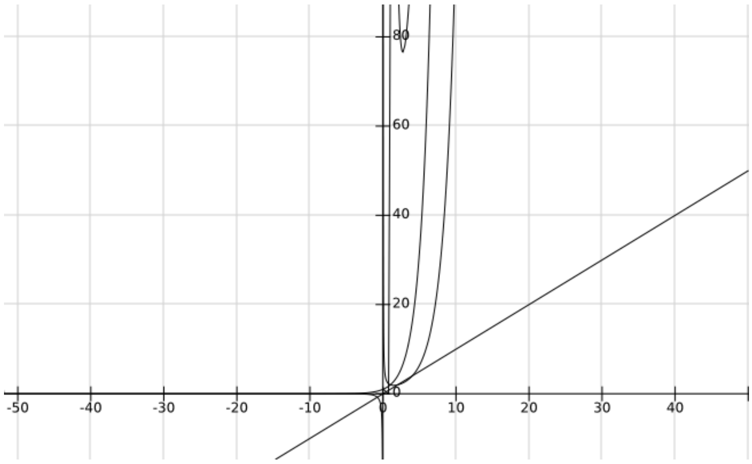

Figure 1 – Graphical plot of PQI-functions

From figure (1) it's seen that PQI-operator is expanding slower than exponential function and, thus, it's reducing to almost linear function O(n).

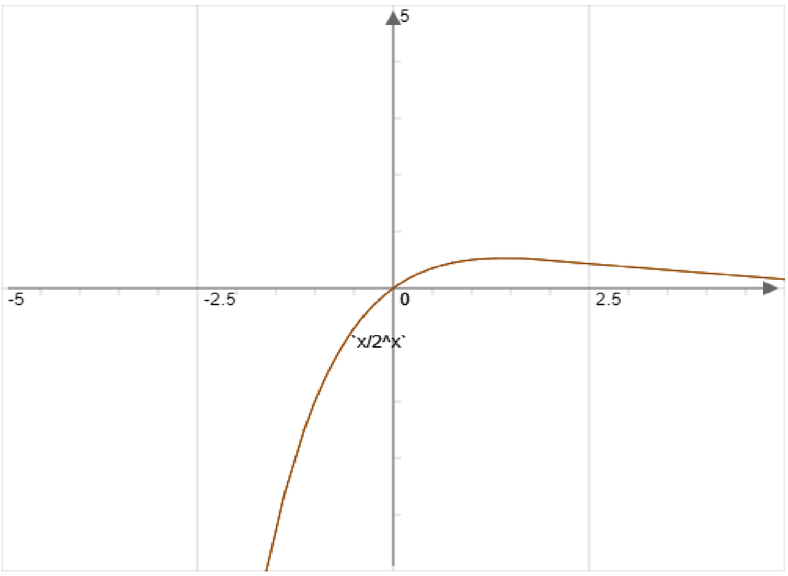

Figure 2 – Graphical plot of PQI-function

The figure (2) gives the fact that the PQI-function is converging to zero and thus has lower orders, rather than when we devise the power of exponential function itself.

Final result for the PQI-operator is derived from the following equlibria:

$$PQI(x)=\frac{2^x}{p(x)}=\{\frac{2^x}{x},2^{\frac{x}{\log(x)}},2^{\log(x)}\}=\frac{f(x)}{p(x)}:O(\frac{f(x)}{p(x)})<O(f(x)). \qquad (14)$$

Conclusion

We have proved that the derivatives of the combinatorial functions are equal with respect to the complexity defined by the big O-notation. From this fact, it follows that there exists the function or PQI-operator which slows down the rapid growth of factorial and power-set functions.

The precise definition of PQI-operator is given by the equation (14). This gives the assumption of the possible ways of reducing the NP-complete problems to the sub-tasks with the slow growth converging to the linear or even logarithmic class of arbitrary degree.

It also follows that the subtraction operator cannot give the expected result of exponential function growth – thus, we have to revert to the class of functions where the relevant changes are made and the growth is reduced and is almost convergent to the linear bounds.

The main outcome from the past works by S. Cook state that NP-complete problems are reducible to the fractions of P-complete problems by applying PQI-derivatives from (1) – (14).

We give more precise note that by introducing the PQI-operator, we are not giving the final outcome for P-NP problem case, however, we are in the state of natural complexity which is, first, proved to be existent in the set of combinatorial functions, then, it's obvious that the Millennium Theorem is rather more disputable on this empirical result.

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ЕСЕПТЕУ КҮРДЕЛІЛІГІ ТЕОРИЯСЫ PQI ОПЕРАТОРЫНА КІРІСПЕ

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Аңдатпа. Бұл мақалада біз PQI операторы ұғымын сипаттаймыз, ол бұрын ішкі жиынның бөлігі ретінде кеңейтілген ақырлы автоматтарды іске асыру үшін қосымша ретінде қолданылған; біз осы оператордың нақты анықтамасын береміз және берілген модельге және алдыңғы жұмысқа сәйкес маңызды күрделілік теоремаларын дәлелдейміз; ұсынылған операция әдісін құрастырылған автоматтар арқылы қайта анықталған күрделілікті бағалау үшін бұрын жақсы зерттелген модификацияланған ішкі жиын арқылы пайдалануға болады. Осылайша, күрделілік күй жарылысына байланысты практикалық емес күрделі алгоритмдердің шешілуін осы зерттеуде бұрын ұсынылған классификацияға сәйкес мысалы, Рабин-Скотт ішкі жиынын құру кезінде әр түрлі болуы мүмкін. Бұл алгоритмдердің күрделілігі факториалдың бір түрі болуы мүмкін. Біз осы жұмыстардың циклінде ұсынылған тәсілден туындайтын PQI операторы бар екенін көрсетеміз, осылайша бұл жұмысты автоматтар теориясы мен күйлердің күрделілігін зерттеудің жалғасы ретінде қарастыруға болады.

Кілттік сөздер: есептеу күрделілігі, оператор, теориялық информатика, ішкі жиындарды құру, ақырғы автоматтар.

ВВЕДЕНИЕ PQI-ОПЕРАТОРА В ТЕОРИЮ ВЫЧИСЛИТЕЛЬНОЙ СЛОЖНОСТИ

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Аннотация. В этой статье мы описываем понятие PQI-оператора, которое использовалось ранее в роли приложения для реализации расширенных конечных автоматов в рамках конструкции подмножества; мы даем точное определение этого оператора и доказываем важные теоремы сложности в соответствии с предоставленной моделью и предыдущей работой; Представленный метод операции может быть использован для оценки переопределенной сложности по автоматам, построенным с помощью модифицированной конструкции подмножества, которая была хорошо изучена ранее. Таким образом, сложность может быть разного типа в соответствии с классификацией, представленной ранее в данном исследовании разрешимости сложных алгоритмов, непрактичных из-за взрыва состояния, например, при построении подмножества Рабина-Скотта. Сложность этих алгоритмов также может быть типом факториала. Мы покажем, что существует PQI-оператор, вытекающий из подхода, представленного в цикле этих работ, таким образом, данную работу можно рассматривать как продолжение изучения теории автоматов и сложности состояний.

Ключевые слова: вычислительная сложность, оператор, теоретическая информатика, построение подмножества, конечные автоматы.

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