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ACCOUNTING FOR THE TEMPERATURE DISTRIBUTION OF A BODY IN THE FORM OF A RECTANGULAR PARALLELEPIPED, TAKING INTO ACCOUNT HEAT TRANSFER USING A VARIATIONAL APPROACH

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Abstract. Finite element method is a numerical method solutions of differential equations, found in physics and technology. The emergence of this method associated with solving problems of space research. It was first published in the work of Turner, Cluj, Martin and Topp. This work contributed to the emergence of other works; a number of articles have been published with examples of the finite element method to the problems of construction mechanics of continuous media. The main idea behind the finite element method is that any continuous quantity such,like temperature pressure and displacement, can be approximated by a discrete model, which is built into a set of piece-continuous functions defined on a finite number of subdomains.

The main idea of the finite element method is that any continuous quantity, such as temperature, pressure and displacement, can be approximated by a discrete model, which is built on a set of piece-continuous functions defined on a finite number of subdomains.

Keywords: Finite element, pipe, motion, cross section, compression, lubrication, system, vibration, equilibrium, continuous value, discrete model, node.

Introduction

In the general case, a continuous quantity is not known in advance, and it is necessary to determine the value of this quantity at some internal points of the region. A discrete model, however, is very easy to construct if one first assumes that the numerical value of this quantity at each interior point of the region is known [1]. After that, we can pass to the general case. So, when constructing a discrete model of a continuous quantity, proceed as follows:

1. A finite number of points are fixed in the area under consideration. These points are called anchor points or simply nodes.

2. The value of the continuous quantity at each nodal point is considered a variable to be defined.

3. The domain of definition of a continuous value is divided into a finite number of subdomains, called elements. These elements have common nodal points and together approximate the shape of the regions.

4. A continuous value is approximated at each element by a polynomial, which is determined using the nodal values of this value. For each element, its own polynomial is determined, but the polynomials are selected in such a way that the continuity of the value along the boundaries of the element would be preserved [2].

The finite element method is based on the idea of approximating a continuous function by a discrete model, which is built on a set of piecewise continuous functions defined on a finite number of subdomains called elements. A polynomial is most often used as a function of an element. The order of the polynomial depends on the number of continuous function data elements used at each node.

Main part

A one-dimensional simplex element is a straight line segment of length L with two nodes, one at each end of the segment [3].

$$T = \varphi_1 T_1 + \varphi_2 T_2$$

$$\varphi_1 = \frac{x_2 - x}{L} \qquad \varphi_2 = \frac{x - x_1}{L}$$

A two-dimensional simplex element is a triangle with straight sides and three nodes, one for each vertex. Logical numbering of element nodes is required.

$$T = \varphi_1 T_1 + \varphi_2 T_2 + \varphi_3 T_3$$

$$\varphi_1 = \frac{1}{2A}(a_1 + b_1 x + c_1 y)$$

$$\begin{cases} a_1 = x_2 y_3 - x_3 y_2 \\ b_1 = y_2 - y_3 \\ c_1 = x_3 - x_2 \end{cases}$$

$$\varphi_2 = \frac{1}{2A}(a_2 + b_2 x + c_2 y)$$

$$\begin{cases} a_2 = x_3 y_1 - y_3 y_1 \\ b_2 = y_3 - y_1 \\ c_2 = x_1 - x_3 \end{cases}$$

$$\varphi_3 = \frac{1}{2A}(a_3 + b_3 x + c_3 y)$$

$$\begin{cases} a_3 = x_1 y_2 - x_2 y_1 \\ b_3 = y_1 - y_2 \\ c_3 = x_2 - x_1 \end{cases}$$

$$2A = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

We consider the three-dimensional simplex element [4]:

$$T = \varphi_1 T_1 + \varphi_2 T_2 + \varphi_3 T_3 + \varphi_4 T_4$$

Consider the three-dimensional function T(x, y, z), whose value is given at the corner points of the parallelepiped (1,2,…8) (1-T, 2-T,…8-T) (Fig. 1)

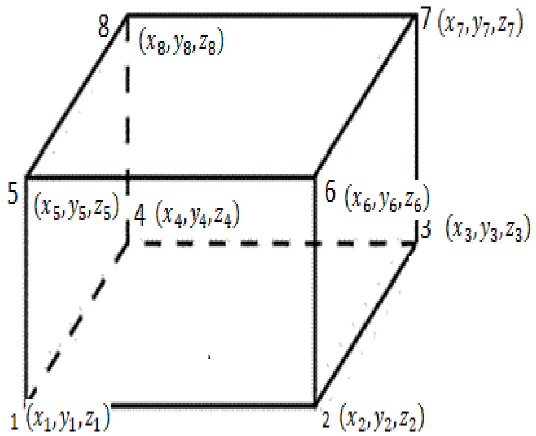

Figure – 1. parallelepiped

If the length of the parallelepiped along the x→ 2a, along the y→ 2b, and along the z→ 2c,then the coordinates of the corner points relative to the center of the parallelepiped will be[5].

1- (-a; в; -c)= (x_1, y_1, z_1)
2- (a; в; -c)= (x_2, y_2, z_2)
3- (a; -в; -c)= (x_3, y_3, z_3)
4- (-a; -в; -c)= (x_4, y_4, z_4)

The value of the function T (x, y, z) to an arbitrary point (x, y, z) inside the parallelepiped isapproximated as follows.

$$\mathrm{T}(x,y,z) = \lambda_1 + \lambda_2 x_1 + \lambda_3 y_1 + \lambda_4 z_1 \qquad (1)$$

To determine the constants A_i (i = 1; 8), we compose the following control system[6]:

$$T_1 = \mathrm{T}(x_1, y_1, z_1);\ T_2 = \mathrm{T}(x_2, y_2, z_2);\ T_3 = \mathrm{T}(x_3, y_3, z_3);$$
$$T_4 = \mathrm{T}(x_4, y_4, z_4); \qquad (2)$$

Substituting the values of the arguments, we get the system of equations:

$$\begin{cases} \lambda_1 + \lambda_2 x_1 + \lambda_3 y_1 + \lambda_4 z_1 = \mathrm{T}_1 \\ \lambda_1 + \lambda_2 x_2 + \lambda_3 y_2 + \lambda_4 z_2 = \mathrm{T}_2 \\ \lambda_1 + \lambda_2 x_3 + \lambda_3 y_3 + \lambda_4 z_3 = \mathrm{T}_3 \\ \lambda_1 + \lambda_2 x_4 + \lambda_3 y_4 + \lambda_4 z_4 = \mathrm{T}_4 \end{cases} \qquad (3)$$

Solving this system of linear equations, we obtain the values of the coefficients $\lambda_1, \lambda_2 \dots \lambda_5$

$$\begin{cases} \Lambda_1 = \frac{\mathrm{T}_8+\mathrm{T}_7+\mathrm{T}_6+\mathrm{T}_5+\mathrm{T}_4+\mathrm{T}_3+\mathrm{T}_2+\mathrm{T}_1}{8} \\ \Lambda_2 = \frac{-\mathrm{T}_8+\mathrm{T}_7+\mathrm{T}_6-\mathrm{T}_5-\mathrm{T}_4+\mathrm{T}_3+\mathrm{T}_2-\mathrm{T}_1}{8\mathrm{a}} \\ \Lambda_3 = \frac{-T_8-T_7+T_6+T_5-T_4-T_3+T_2+T_1}{8b} \\ \Lambda_4 = \frac{\mathrm{T}_8+\mathrm{T}_7+\mathrm{T}_6+\mathrm{T}_5-\mathrm{T}_4-\mathrm{T}_3-\mathrm{T}_2-\mathrm{T}_1}{8\mathrm{c}} \end{cases} \qquad (7)$$

(7) substituting these values into equation (2) we obtain:

$$\mathrm{T}(x, y, z) = \varphi_1(x,y,z) * T_1 + \varphi_2(x,y,z) * T_2 + \varphi_3(x,y,z) * T_3 + \varphi_4 T_4 + \varphi_5(x,y,z) * T_5 + \varphi_6(x,y,z) * T_6 + \varphi_7(x,y,z) * T_7 + \varphi_8(x,y,z) * T_8; \qquad (4)$$

-a ≤ x ≤a; -b ≤x ≤b; -c ≤ x≤ c;

Here φ_i(i= 1, 8) are defined as follows: [7]

$$\varphi_1(x, y, z) = (\tfrac{1}{8} - \tfrac{x}{8a} + \tfrac{y}{8b} - \tfrac{z}{8c} - \tfrac{xy}{8ab} + \tfrac{xz}{8ac} - \tfrac{yz}{8bc} + \tfrac{xyz}{8abc});$$
$$\varphi_2(x, y, z) = (\tfrac{1}{8} + \tfrac{x}{8a} + \tfrac{y}{8b} - \tfrac{z}{8c} + \tfrac{xy}{8ab} - \tfrac{xz}{8ac} - \tfrac{yz}{8bc} - \tfrac{xyz}{8abc});$$
$$\varphi_3(x, y, z) = (\tfrac{1}{8} + \tfrac{x}{8a} - \tfrac{y}{8b} - \tfrac{z}{8c} - \tfrac{xy}{8ab} - \tfrac{xz}{8ac} + \tfrac{yz}{8bc} + \tfrac{xyz}{8abc});$$
$$\varphi_4(x, y, z) = (\tfrac{1}{8} - \tfrac{x}{8a} - \tfrac{y}{8b} - \tfrac{z}{8c} + \tfrac{xy}{8ab} + \tfrac{xz}{8ac} + \tfrac{yz}{8bc} - \tfrac{xyz}{8abc}); \qquad (5)$$
$$\varphi_5(x, y, z) = (\tfrac{1}{8} - \tfrac{x}{8a} + \tfrac{y}{8b} + \tfrac{z}{8c} - \tfrac{xy}{8ab} - \tfrac{xz}{8ac} + \tfrac{yz}{8bc} - \tfrac{xyz}{8abc});$$
$$\varphi_6(x, y, z) = (\tfrac{1}{8} + \tfrac{x}{8a} + \tfrac{y}{8b} + \tfrac{z}{8c} + \tfrac{xy}{8ab} + \tfrac{xz}{8ac} + \tfrac{yz}{8bc} + \tfrac{xyz}{8abc});$$
$$\varphi_7(x, y, z) = (\tfrac{1}{8} + \tfrac{x}{8a} - \tfrac{y}{8b} + \tfrac{z}{8c} - \tfrac{xy}{8ab} + \tfrac{xz}{8ac} - \tfrac{yz}{8bc} - \tfrac{xyz}{8abc});$$
$$\varphi_8(x, y, z) = (\tfrac{1}{8} - \tfrac{x}{8a} - \tfrac{y}{8b} + \tfrac{z}{8c} + \tfrac{xy}{8ab} - \tfrac{xz}{8ac} - \tfrac{yz}{8bc} + \tfrac{xyz}{8abc});$$

why -a ≤ x ≤a; -b ≤y ≤b; -c ≤ z≤ c.

The value of the functions φ_i(x, y, z) (i=1,8) at the corner points of the parallelepiped isdetermined as follows[8]:

$\varphi_1(x_1, y_1, z_1)$= 1; $\varphi_1(x_1, y_1, z_1) = \varphi_1(x_2, y_2, z_2) = \varphi_1(x_3, y_3, z_3) = \varphi_1(x_4, y_4, z_4) = \varphi_1(x_5,$

y_5, z_5)= $\varphi_1(x_6, y_6, z_6)$= $\varphi_1(x_7, y_7, z_7)$= $\varphi_1(x_8, y_8, z_8)$=0

$\varphi_2(x_2, y_2, z_2)$= 1; $\varphi_2(x_1, y_1, z_1)$= $\varphi_2(x_2, y_2, z_2)$= $\varphi_2(x_3, y_3, z_3)$= $\varphi_2(x_4, y_4, z_4)$= $\varphi_2(x_5, y_5, z_5)$= $\varphi_2(x_6, y_6, z_6)$= $\varphi_2(x_7, y_7, z_7)$= $\varphi_2(x_8, y_8, z_8)$=0

$\varphi_3(x_3, y_3, z_3)$= 1; $\varphi_3(x_1, y_1, z_1)$= $\varphi_3(x_2, y_2, z_2)$= $\varphi_3(x_3, y_3, z_3)$= $\varphi_3(x_4, y_4, z_4)$= $\varphi_3(x_5, y_5, z_5)$= $\varphi_3(x_6, y_6, z_6)$= $\varphi_3(x_7, y_7, z_7)$= $\varphi_3(x_8, y_8, z_8)$=0

$\varphi_4(x_4, y_4, z_4)$= 1; $\varphi_4(x_1, y_1, z_1)$= $\varphi_4(x_2, y_2, z_2)$= $\varphi_4(x_3, y_3, z_3)$= $\varphi_4(x_4, y_4, z_4)$= $\varphi_4(x_5, y_5, z_5)$= $\varphi_4(x_6, y_6, z_6)$= $\varphi_4(x_7, y_7, z_7)$= $\varphi_4(x_8, y_8, z_8)$=0

$\varphi_5(x_5, y_5, z_5)$= 1; $\varphi_5(x_1, y_1, z_1)$= $\varphi_5(x_2, y_2, z_2)$= $\varphi_5(x_3, y_3, z_3)$= $\varphi_5(x_4, y_4, z_4)$= $\varphi_5(x_5, y_5, z_5)$= $\varphi_5(x_6, y_6, z_6)$= $\varphi_5(x_7, y_7, z_7)$= $\varphi_5(x_8, y_8, z_8)$=0

$\varphi_6(x_6, y_6, z_6)$= 1; $\varphi_6(x_1, y_1, z_1)$= $\varphi_6(x_2, y_2, z_2)$= $\varphi_6(x_3, y_3, z_3)$= $\varphi_6(x_4, y_4, z_4)$= $\varphi_6(x_5, y_5, z_5)$= $\varphi_6(x_6, y_6, z_6)$= $\varphi_6(x_7, y_7, z_7)$= $\varphi_6(x_8, y_8, z_8)$=0

$\varphi_7(x_7, y_7, z_7)$= 1; $\varphi_7(x_1, y_1, z_1)$= $\varphi_7(x_2, y_2, z_2)$= $\varphi_7(x_3, y_3, z_3)$= $\varphi_7(x_4, y_4, z_4)$= $\varphi_7(x_5, y_5, z_5)$= $\varphi_7(x_6, y_6, z_6)$= $\varphi_7(x_7, y_7, z_7)$= $\varphi_7(x_8, y_8, z_8)$=0

$\varphi_8(x_8, y_8, z_8)$= 1; $\varphi_8(x_1, y_1, z_1)$= $\varphi_8(x_2, y_2, z_2)$= $\varphi_8(x_3, y_3, z_3)$= $\varphi_8(x_4, y_4, z_4)$= $\varphi_8(x_5, y_5, z_5)$= $\varphi_8(x_6, y_6, z_6)$= $\varphi_8(x_7, y_7, z_7)$= $\varphi_8(x_8, y_8, z_8)$=0 (6)

Conclusion

In the result, there is obtained common variation functional for defining the law of temperature distribution in the body of rectangular 3D cube form. Results of this work can be used to determine the law of temperature distribution in three-dimensional rods in the form of a parallelepiped. There is executed practical implementation of the developed approach as a concrete example, when heat flux is directed to one of 3D cube's facets, and on the opposite facet there occurs heat exchange with the environment. There was shown temperature values proximity of corresponding nodes of rectangular 3D cube upon separation of its facets for one, two and three intervals.

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ВАРИАЦИЯЛЫҚ ТӘСІЛДІ ПАЙДАЛАНҒАН ЖЫЛУ ТАРТЫЛУЫН ЕСКЕ АЛУ МЕН ТӨРТ БҰРЫШТЫ ПАРАЛЛЕЛЕППЕТ ПІШІНДЕГІ ДЕНЕ ТЕМПЕРАТУРАСЫНЫҢ БӨЛУІНЕ ЕСЕП

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Accounting for the temperature distribution of a body in the form of a rectangular parallelepiped, taking into account heat transfer using a variation al approach

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Аңдатпа. Ақырлы элементтер әдісі – физика мен техникада кездесетін дифференциалдық теңдеулерді шешудің сандық әдісі. Бұл әдістің пайда болуы ғарышты зерттеу мәселелерін шешумен байланысты. Ол алғаш рет Тернер, Клуж, Мартин және Топптың жұмысында жарияланған. Бұл еңбек басқа еңбектердің пайда болуына ықпал етті; Үздіксіз ортаның құрылыс механикасы мәселелеріне соңғы элементтер әдісінің мысалдарымен бірқатар мақалалар жарияланды. Ақырлы элементтер әдісінің негізгі идеясы - температура қысымы және орын ауыстыру сияқты кез келген үздіксіз шаманы ішкі домендердің шектеулі санында анықталған үзіліссіз функциялар жиынтығына кіріктірілген дискретті модель арқылы жуықтауға болады.

Ақырлы элементтер әдісінің негізгі идеясы температура, қысым және орын ауыстыру сияқты кез келген үздіксіз шаманы ішкі домендердің шектеулі санында анықталған үзіліссіз функциялар жиынтығына құрылған дискретті модель арқылы жуықтауға болады.

Түйін сөздер: Ақырлы элемент, құбыр, қозғалыс, қима, қысу, майлау, жүйе, діріл, тепе-теңдік, үздіксіз шама, дискретті модель, түйін.

УЧЕТ РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУРЫ ТЕЛА В ВИДЕ ПРЯМОУГОЛЬНОГО ПАРАЛЛЕПИПЕДА С УЧЕТОМ ТЕПЛООТДАЧИ С ИСПОЛЬЗОВАНИЕМ ВАРИАЦИОННОГО ПОДХОДА

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Абстракт. Метод конечных элементов представляет собой численный метод решения дифференциальных уравнений, встречающихся в физике и технике. Возникновение этого метода связано с решением задач космических исследований. Впервые он был опубликован в работе Тернера, Клужа, Мартина и Топпа. Эта работа способствовала появлению других работ; опубликован ряд статей с примерами применения метода конечных элементов к задачам строительной механики сплошных сред. Основная идея метода конечных элементов заключается в том, что любая непрерывная величина, такая как температура, давление и перемещение, может быть аппроксимирована дискретной моделью, которая встроена в набор кусочно-непрерывных функций, определенных в конечном числе подобластей.

Основная идея метода конечных элементов состоит в том, что любую непрерывную величину, такую как температура, давление и перемещение, можно аппроксимировать дискретной моделью, построенной на наборе кусочно-непрерывных функций, заданных

на конечном числе подобластей.

Ключевые слова: Конечный элемент, труба, движение, сечение, сжатие, смазка, система, вибрация, равновесие, непрерывная величина, дискретная модель, узел.

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